

# Cosmic Web: skeleton, connectivity, non-gaussianity

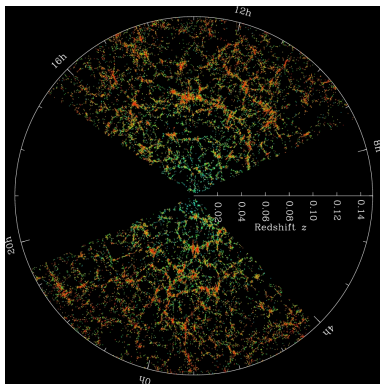
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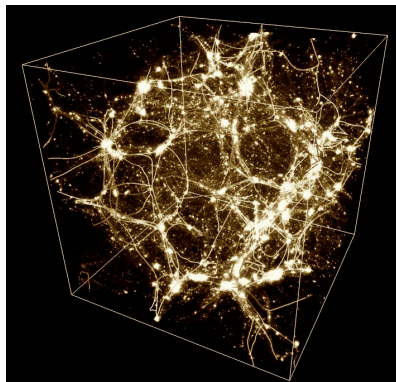
*November 2, 2016*

with: S. Codis, C. Pichon, (IAP),  
T. Sousbie, C. Gay, Dick Bond, L. Kofman . . .

## Large scale filamentary structure connecting clusters of galaxies is evident both in data and simulations



Sloan DS Survey

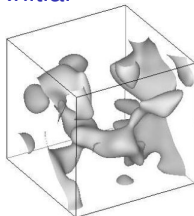


Horizon (IAP) project

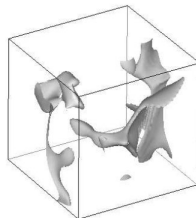
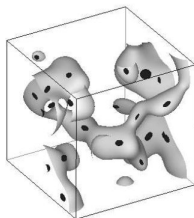
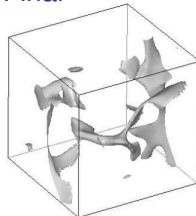
# From Peaks to Filaments, Skeleton of the Cosmic Web

- The high rare peaks of the density field largely define how large scale structure looks like. Filaments are dense bridges between the peaks
- **The Skeleton of LSS** traces the Cosmic Web of filaments and provides the next level of detail to LSS understanding.
- At large scales, peaks and filaments can be traced back to initial density field

Initial



Final



# Geometry of cosmological density field

## Famous works

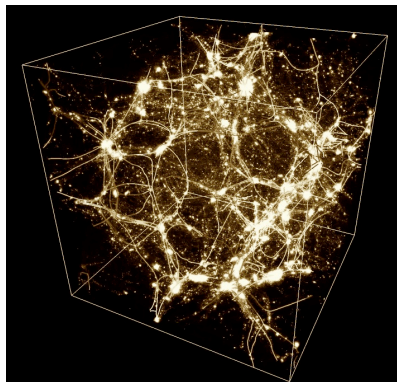
Doroshkevich, 1970

Arnold, Zeldovich, Shandarin, 1982

Bond, Bardeen, Kaiser, Szalay, 1986

## Geometrical questions:

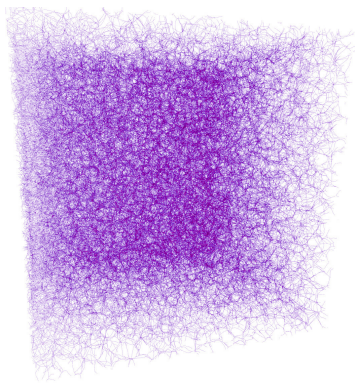
- abundance and shapes of peaks of different scale
- Length of filamentary bridges
- Connectivity of peaks by filamentary ridges
- Relation between the properties of filaments and clusters
- Percolation characteristics of filamentary network



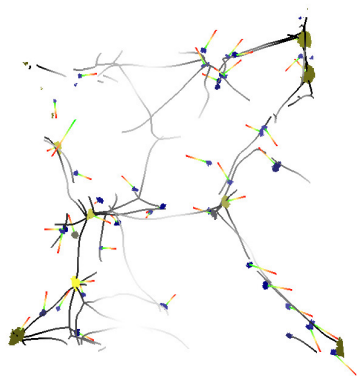
Sims: Horizon project + skeleton  
(DisPerSE, T. Sousbie 2011)

Skeleton - geometrical description  
of filaments

## Skeleton as tracer of the filaments



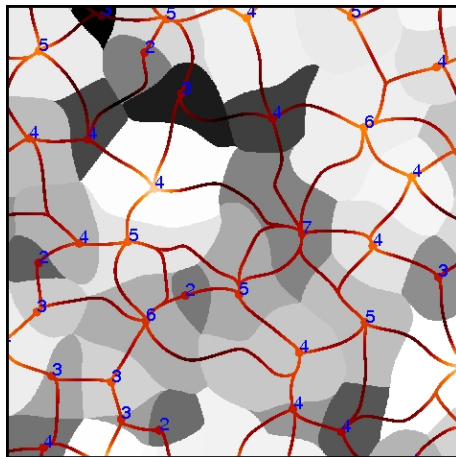
Dark matter skeleton in  
Horizon  $4\pi$  simulation  
T. Sousbie et al, 2011



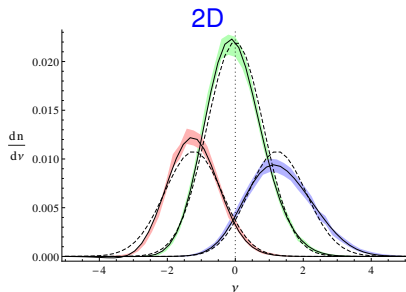
Halo spin wrt the skeleton  
Codis et al, 2012, Noam Libeskind,  
Elmo Tempel, Stefan Gottlöber  
Tidal torque near filaments  
Codis et al, 2015

# Fully connected Global Skeleton

- skeleton is a set of gradient lines connecting peaks
- skeleton line between peaks passes through one saddle point
- **Connectivity**  $N_c$  is number of peaks the given peak is connected to
- $N_c$  as the function of peak properties, and/or restriction on saddles is the main object
- Properties of saddles are critical for percolation
- but not every segment of the skeleton is a dense filament



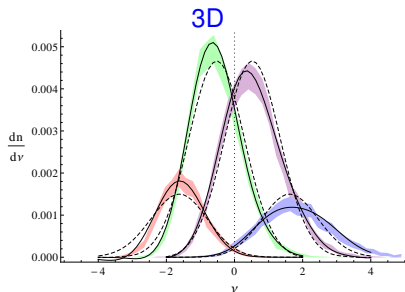
# Basic info on Gaussian Peaks and Extrema Counts (BBKS)



$$\langle n_{\text{peak}} \rangle = \frac{1}{8\sqrt{3}\pi R_*^2}, \quad \langle n_{\text{saddle}} \rangle = \frac{1}{4\sqrt{3}\pi R_*^2}$$

$$\bar{R}_{\text{peak-to-peak}} \approx 6.6 R_* = 6.6 \frac{\sigma_1}{\sigma_2}$$

$$\frac{\langle n_{\text{saddle}} \rangle}{\langle n_{\text{peak}} \rangle} = 2 \Rightarrow \langle N_c \rangle = 4$$

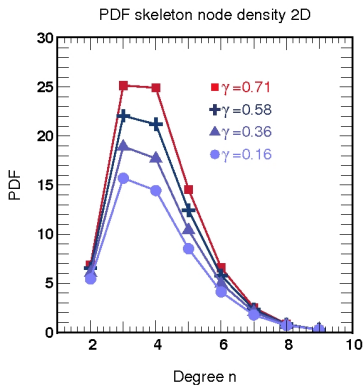


$$\langle n_{+-} \rangle = \frac{29\sqrt{15} \mp 18\sqrt{10}}{1800\pi^2 R_*^3}$$

$$\bar{R}_{\text{peak-to-peak}} \approx 6.8 R_* = 6.8 \frac{\sigma_1}{\sigma_2}$$

$$\frac{\langle n_{\text{saddle}} \rangle}{\langle n_{\text{peak}} \rangle} \approx 3.055 \Rightarrow \langle N_c \rangle = 6.1 ?$$

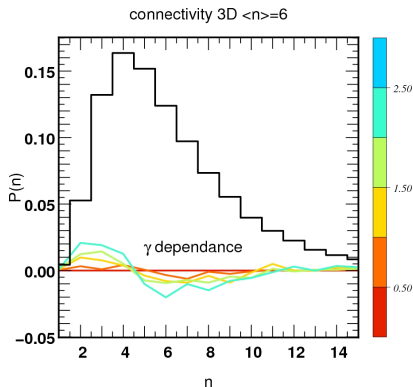
# Connectivity of the Global Skeleton



**Notable Result:**

2D  $\langle N_c \rangle = 4$  - average connections of a peak

3D  $\langle N_c \rangle = 6$  - average connections of a peak



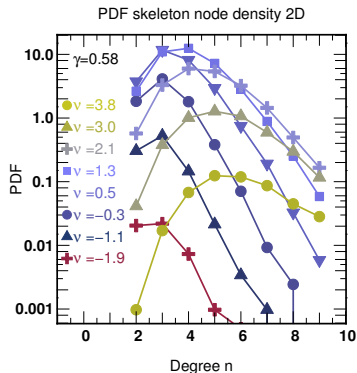
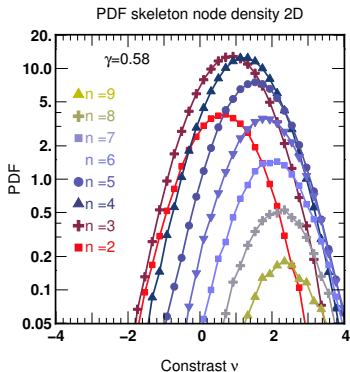
**Matches well with**

$2\langle n_{sad} \rangle / \langle n_{max} \rangle = 4$

$2\langle n_{sad} \rangle / \langle n_{max} \rangle \approx 6.1$



# $\nu$ dependence of the connectivity



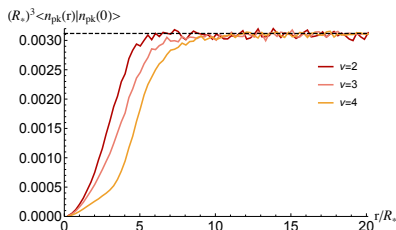
## Notable Result:

High peaks tend to have more connections

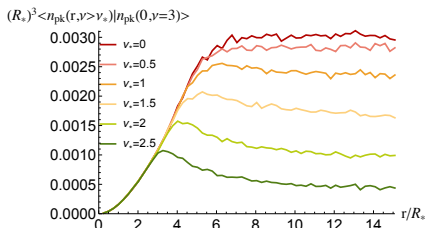
Peaks with large number of connections are predominantly high

# Towards connectivity theory, $N_c$

Idea: Count the number of saddles up to  $R$  . . . , conditional on the properties of the peak. But what is  $R$  ? Some characteristic distance to the next peak. Let us look at a peak with height  $\nu = \delta / \sigma = 2.5$ . How far is the next peak ? Compute conditional peak densities and define  $R$  as the most probable distance to the next peak



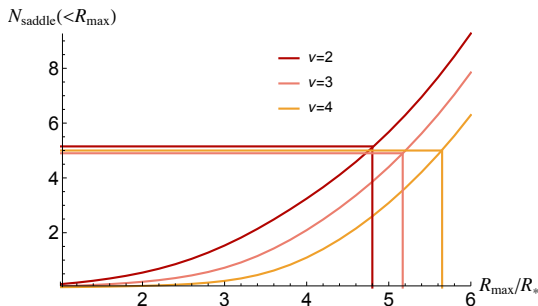
if to **any other** peak,  $R \geq \bar{R}_{\text{peak-to-peak}}$   
and increases with  $\nu$



if to **other high** peak,  $R < \bar{R}_{\text{peak-to-peak}}$   
(Kaiser bias)

# $N_c \approx$ counting saddles to the next peak

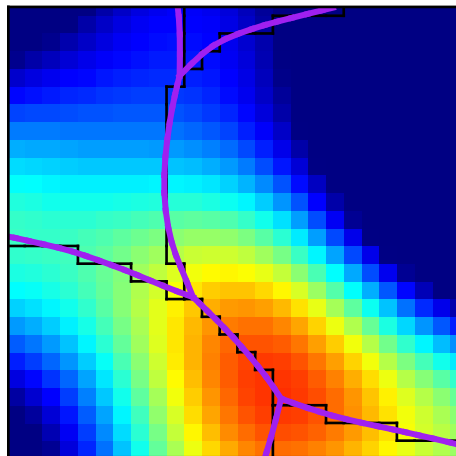
Number of saddles to distance  $R$  conditional on the height  $\nu$  of central peak



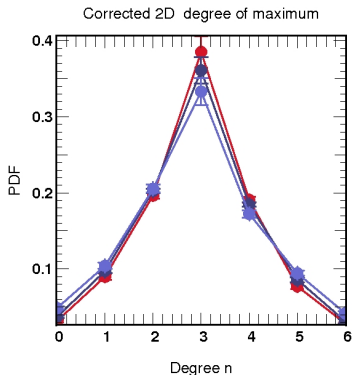
- Lower density of saddles around higher peaks are compensated by larger distances to the next peak.  $N_c$  remain relatively insensitive to  $\nu$  ? Is there an inconsistency with measurements ?

## Local Skeleton

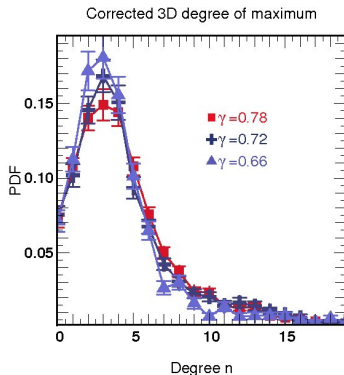
- Near the peak real filaments follows ridges of the density
- Task is to count the number of filaments  $N_f$  that leave a peak.
- Filament structure reflects merger history and flow pattern around in the peak-patch.
- Depends on distance, (very) locally each peak is elliptical and has two ridges leaving it.
- Thus (coarse grained) skeleton line bifurcate,  $N_f = N_c - N_{bifur}$ .
- frequent close bifurcations - link to catastrophe theory ?



## Local number of filaments $N_f$ near the peak



$$2D \langle N_f \rangle = 3$$

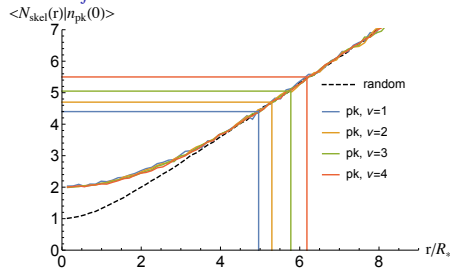


$$3D \langle N_f \rangle = 4$$

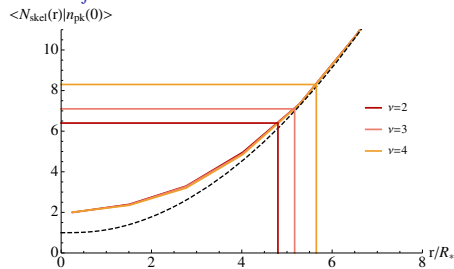
- Notable Results: average number of branches from a peak
- Interesting link: [Adhesion model](#) that enforces these values by construction
- $N_{\text{bifurcations}} = N_c - N_f$
- Issues: not all branches have high density and are physically important

# Counting filaments as maxima on a sphere around the peak

## 2D, $N_f(r)$



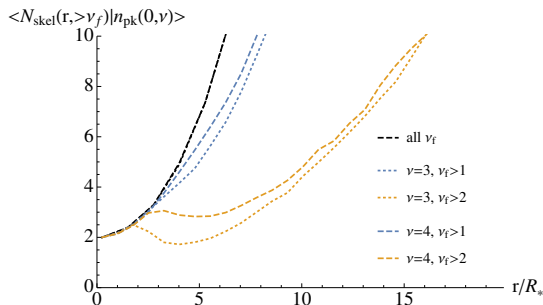
## 3D, $N_f(r)$



## Preliminary conclusions

- At  $r \sim 0$   $N_f = 2$  if we have peak at the center.  $N_f = 1$  otherwise
- $N_f(r)$  does not depend on peak height, but distance to the next peak does, which explains dependence of connectivity on height

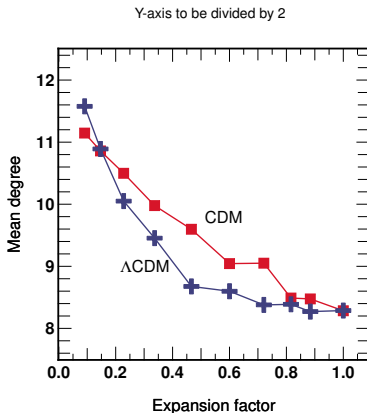
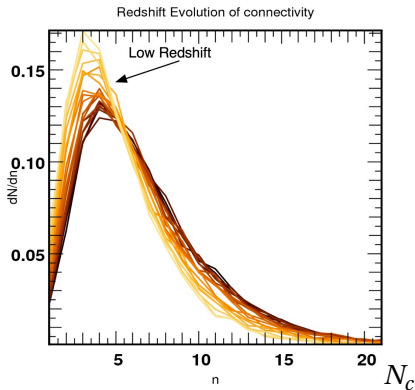
# Not all filaments are equally prominent. Counting important ones



- Number of dense  $\nu_f > 2$  filamentary bridges is increasing with the height of the central peak
- Not very rare  $\nu = 3$  central peak has two (branches of) dense filaments, i.e it sits in one dominant filament on average
- Rare  $\nu = 4$  peak is at intersection of three prominent bridges.

# Connectivity of a non-Gaussian field differ from the Gaussian

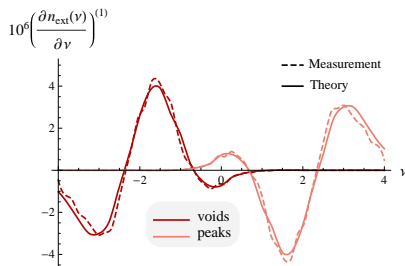
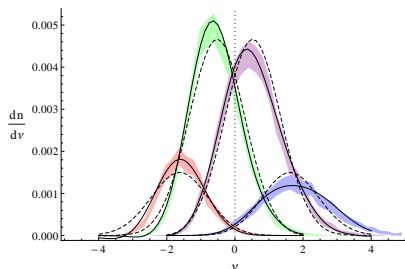
- In cosmological simulations, as density becomes more non-Gaussian, connectivity of the Cosmic Web decreases
- This leads to model dependent history of the connectivity at different redshifts.





# Non-Gaussian 3D Extrema Counts (Gay et al, 2011)

$$\langle n_{\mp--} \rangle = \frac{29\sqrt{15} \mp 18\sqrt{10}}{1800\pi^2 R_*^3} + \frac{5\sqrt{5}}{24\pi^2 \sqrt{6\pi} R_*^3} \left( \langle q^2 J_1 \rangle - \frac{8}{21} \langle J_1^3 \rangle + \frac{10}{21} \langle J_1 J_2 \rangle \right)$$



At  $\sigma \approx 0.2$

$$\frac{\langle n_{saddle} \rangle}{\langle n_{peak} \rangle} \approx 2.5 \Rightarrow N_c \approx 5$$

(cubic moments evaluate, with some spectral dependence, to  $\approx 0.1$ , see Gay, Pichon, Pogosyan, 2011)

## Summary and what is left behind the scenes

- Description of the filamentary Cosmic Web poses many questions that can be formulated in geometrical or topological language.
- This allows for novel computation and powerful analytical technique as well as methods of analysis of simulations and data.
- Skeleton, in different version, is one such technique that has been proven extremely helpful 'marking' large scale structure for study its effect, for example, on galaxy properties and formation (not in this talk).
- In this talk we focused on connectivity of the skeleton of the Cosmic Web, which allowed as to start formulate analytical explanation of how it works.
- We found evidence of increased connectivity for rare peaks - i.e more massive galaxy clusters.
- Evolving cosmic density field becomes non-Gaussian as non-linearity develops. This affects prominence of the filaments and connectivity of the Web. Different cosmological models, as the result, has different history of connectivity which may be observed.
- Formalism of geometrical measures in non-gaussian regime that we developed (not in this talk) can provide the basis for analysing (mildly) non-linear skeleton. Simple example of direct extrema counts support simulation measurements on decrease of connectivity with non-linearity.

## Cosmic Web in 2D Adhesion Model

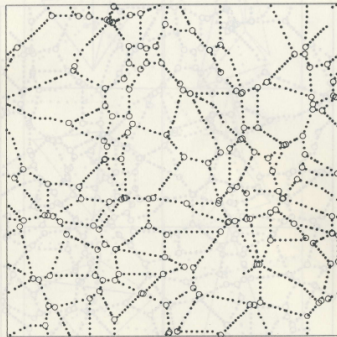


Fig.9b

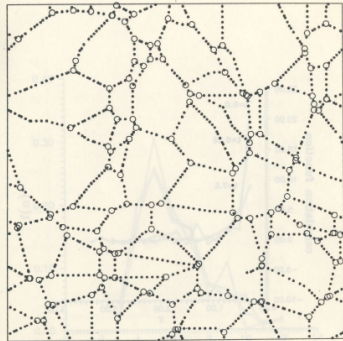
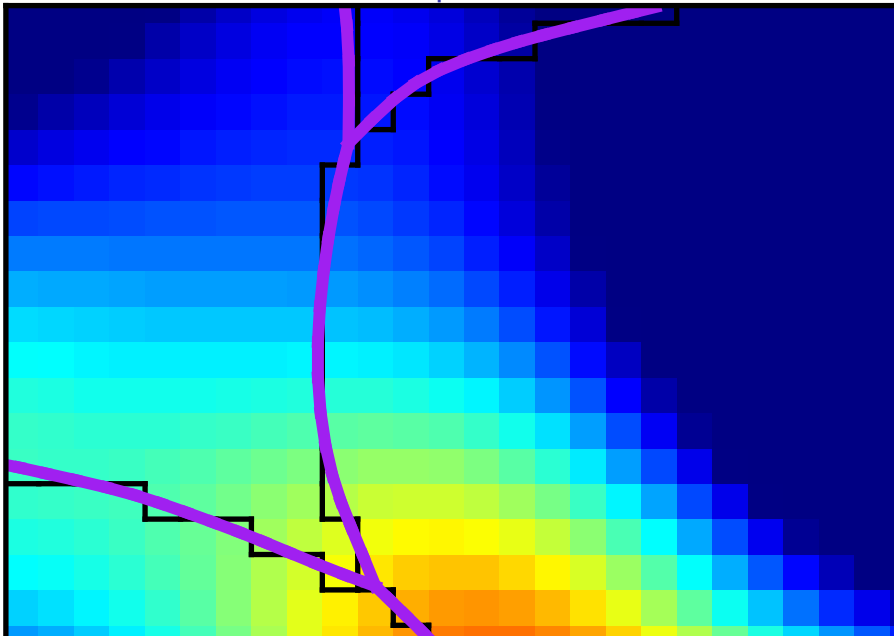
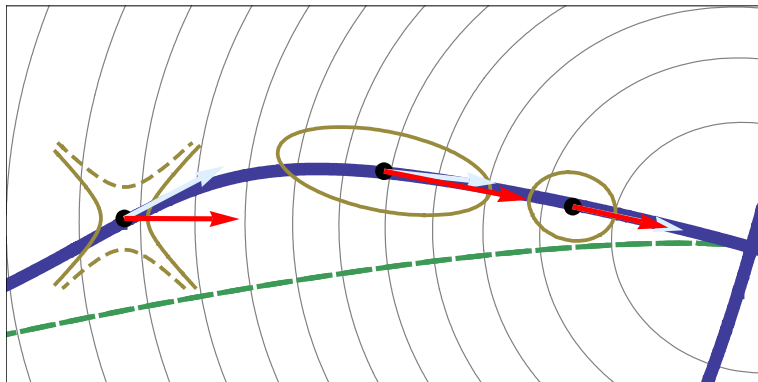


Fig.9c

## Towards the local description of the Skeleton



# Towards the local description of the Skeleton



$$(\nabla \nabla \rho) \cdot \nabla \rho = \lambda \nabla \rho$$